

Cognitive Skills and Math Performance

While reading difficulties have been studied extensively in the last few decades, difficulties learning math have received less attention. Math disabilities, however, are as common as reading disabilities at 5 to 8% of the population¹. Like reading disabilities, math disabilities may have multiple causes and dimensions. Moreover, as the math curriculum has changed over the last ten to fifteen years, more emphasis has been placed on communicating math ideas, interpreting data from charts and figures, and estimating. These changes mean that learning deficits in any area are more likely to impact performance in math. In fact, many individuals with a math disability also have learning disabilities in other areas. Whether or not other learning disabilities are present, however, there is growing evidence that underlying cognitive processes play a significant role in math performance.²

Geary describes the relationship between various cognitive mechanisms and deficits in math ability which may be summarized in the following chart:

Cognitive mechanisms	Math deficit
Language systems	Information representation, as in articulating number words.
Working memory	Information manipulation as during the act of counting.
Visual-spatial processing	Representations of conceptual knowledge, such as number magnitudes and information in a spatial form (as in a chart).
Attentional and inhibitory processes (executive controls)	Using procedures during problem solving.

Based on an analysis of the mechanisms and related types of math deficits for which there is empirical support, Geary defines three subtypes of math disability: “Semantic Memory”, “Procedural” and “Visual-Spatial.” The following discussion provides an overview of the three subtypes, the deficits associated with them and the underlying cognitive skills impairing effective performance.

Semantic Memory Subtype

Semantic memory is what enables us to remember words, meanings and concepts, as opposed to specific events. Individuals with deficits in semantic memory are likely to have difficulty retrieving arithmetic facts, manifesting a high error rate and/or a variable (often slow) retrieval rate. These individuals may also fail to recognize math symbols (e.g., operations signs) and may have trouble grasping the meaning of different operations. It would not be surprising to find a student with a semantic memory disability doing a series of math problems with a mixture of operations (e.g., a combination of addition and subtraction problems) all as one kind of problem, because he/she did not notice the change in the operations sign from one problem to the next.

¹ Stephen Zecker, presentation at AET, February 2008.

² David C. Geary. “Mathematics and Learning Disabilities” *Journal of Learning Disabilities*. 37(1), 2004; 4-1

This subtype is frequently co-morbid with a reading or language processing disability, as it requires reliance on phonetic and semantic representational systems.

In a normally developing individual, basic math fact retrieval has been suggested to progress from counting to direct fact retrieval. For example, for the basic fact:

$$4+6=?$$

First we learn by “counting all”. This involves counting every number up to the sum, as in “1, 2, 3, 4 ... 1, 2, 3, 4, 5, 6 ... 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.”

Then we move to “counting on”. In this step, the student starts with the first addend and counts up through the second, as in “4, 5, 6, 7, 8, 9, 10.”

A more mature version of “counting on” involves choosing the larger number and counting from there, as in “6, 7, 8, 9, 10.” This is referred to as a minimization approach.

Finally these processes become so automatic that we move to the fastest and most efficient retrieval of facts known as “direct retrieval”. When a complement of math facts reside in long-term memory, the individual sees “4 + 6 =” and immediately retrieves the answer, “10.”

Most individuals figure out the first three steps without direct instruction, whereas the direct retrieval of facts generally requires direct instruction. The traditional method of teaching math facts through direct instruction involves extensive drilling. However, when a student has a Semantic Memory subtype of math disability, it has been difficult to remediate with traditional methods (drills).

Some educators recommend circumventing the problem and removing the burden of the retrieval process. This could involve using a math facts “cheat sheet” or a calculator. However, this approach does nothing to remedy the underlying retrieval problems that give rise to the problem in the first place.

Since the characterization of this subtype refers to various aspects of memory, including short-term memory and working memory, it is not surprising that remediation through traditional approaches is often ineffective. This begs the question whether there is a way to more effectively remediate semantic memory deficits by developing the mind’s underlying ability to develop concepts and the facilities of symbolic representation (long-term memory) and working memory rather than simply circumventing the problem with the “crutch” of a “cheat sheet.”

Procedural Subtype

The Procedural subtype of math disability relates to the use of strategies and procedures to solve math problems. Individuals with this type of disability manifest developmentally immature procedures in solving math problems, have difficulty executing procedures accurately, and difficulty sequencing steps in a procedure. Since procedures are one type of long-term memory, difficulty or slowness in identifying and working through the execution of a procedure are also likely to be memory issues, albeit of a different type. Being able to identify a pattern, discriminate similarities and differences, and execute a familiar series of steps in a procedure relate directly to underlying cognitive skills, such

as sequential visual or auditory memory, working memory, logic and reasoning, and, of course, attention skills.

Visual-Spatial Subtype

The third subtype of math disability for which there is empirical evidence is the Visual-Spatial subtype. Difficulties in this arena include misaligning numbers in multi-column arithmetic, reversing or putting numbers out of sequence and misunderstanding spatially represented numbers (e.g., exponents). Interpreting charts and graphs and developing an understanding of depicted geometric figures would be other examples of math disabilities of the visual-spatial subtype. Directionality, visualization, timing/rhythm and visual-spatial memory are basic cognitive skills that form the foundation for these specific math-related skills.

Core Deficits in Math Disabilities

While research supports the existence of subtypes of math disability, the discussion above starts to suggest that there may be core deficits underlying the subtypes, just as core deficits have been identified for dyslexia and other reading disabilities. The impact of deficits in underlying cognitive skills may differ from subtype to subtype, but the following common core deficits have been identified³:

- Strategy development
- Working memory
- Retrieval
- Conceptual knowledge
- Speed of processing (fluency)
- Language processing
- Counting knowledge

The following chart shows the connection between basic cognitive skills and math performance:

³ Zecker, op. cit.

Cognitive Skill	How the skill relates to math
Visual Sustained Attention	Staying focused for a period of time – even just long enough to accumulate several digits into the correct order or several numbers and operators into a mathematical sentence.
Visual Discrimination	The ability to differentiate a “+” from a “X” and an “5” from a “2” – and to recognize them quickly without consciously having to evaluate them. Likewise, the ability to see differences in a chart or graph.
Visual Form Consistency	The ability to recognize an object regardless of its size, distance or orientation is necessary in understanding and seeing relationships in geometry and other math concepts.
Visual Processing Speed	The faster the brain can process the visual stimuli that appear on the page, the more likely it is that one can combine them into meaningful groups.
Visualization	The ability to recall an image of what has been seen and the ability to mentally manipulate and change aspects of that image in the mind is inherent in “seeing” what would happen when 2 of 10 apples were removed, for example.
Directionality	Directionality refers to the ability to interpret and project the concepts of “left” and “right” into space and onto other objects. Arguably this is a primary cognitive skill involved in having a mental representation of a number line. Directionality combined with other cognitive skills would also be involved in seeing place value and aligning columns of numbers.
Long-Term Memory	The ability to store and retrieve information, particularly the meaning of concepts and symbols, as well as math facts, is critical to math fluency.
Auditory/Visual Sequential Memory	The ability to recall a sequence of steps in a math procedure is critical in remembering and accurately executing the steps.
Working Memory	Being able to hold information in your mind while you manipulate it – that is, think about it – is what enables a student to assemble math facts and manipulate them at the same time, as well as to hold his/her place in the steps of a procedure.
Conceptual Thinking	One aspect of abstract or interpretive thinking is the ability to form concepts – collections of features that create a category or an idea. The concept of number is core to math ability, but other concepts such as addition/subtraction, cardinality, angles and the like are also integral to math learning.

The Cognitive Foundation for Counting and Number Sense

While there is some disagreement about what number sense actually is, there is broad agreement that it is as vital to math performance as phonemic awareness is to reading⁴. In the same way that beginning readers must recognize that words are composed of sounds which are abstract units (phonemic awareness), math competency is founded on a sense of numbers as discrete but connected entities in a sequence. The issue, much like phonemic awareness in reading, is one of automaticity. Because the human mind has a limited ability to process information consciously, basic facts must become automatic. If, for example, an individual needs to work consciously on combining sounds into a word or to conclude that $7+8$ equals 15 fifteen (such as counting on fingers), then little room or energy is left for understanding concepts. Development of

⁴ Russell Gersten and David J. Chard, Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities, Idoneline.org., 2001

number sense is an important precursor of formal math instruction and includes such elements as

- Counting
- Numerical relations (magnitude comparisons)
- Number transformation
- Estimation
- Number patterns
- Mental number line

The concepts of numbers and their sequential relationships are central to performance in math. In fact, early facility with counting is associated with strong calculation skills later. While counting may seem simple, it comprises several sub-skills, including:

- The assignment of names to the numbers. In this endeavor, English with its “eleven” as compared to Chinese’s “ten one” is problematic.
- An understanding that you count each item once and only once, known as one-to-one correspondence.
- The ability to keep track of which items have been counted and which haven’t yet, a skill known as partitioning.
- The concept of cardinality. This is the notion that the last number counted is the quantity of the counted things.
- An understanding that dissimilar objects can be counted, known as abstraction. That is, you can count apples and oranges (pieces of fruit). It also includes understanding that you can count intangible things.

An examination of the way we develop the concept of number and “number sense” begins to yield some insights into the importance of cognitive skills in the process. Recent research indicates that the concept of number develops early. Nine-month-old infants demonstrate the ability to differentiate between groups of different sizes. The speculation is that a timing mechanism is at work – the time it takes to scan a larger (larger in number, not in size of objects) group is longer than a smaller group. This suggests that timing and rhythm may also be underlying cognitive skills that account for observations that music and math ability are strongly related.

In the 1980s, when educators concluded that students needed to develop automaticity with math facts so that attention resources could be devoted to higher-order thinking, math drills became a common approach in order to drive math facts into Semantic memory. This “drill and kill” approach did produce some results on performance on tests, but did little for the development of number sense.

An alternative approach to developing automaticity is to develop basic cognitive skills such as those described above, including visualization, visual form consistency, visual sequential memory and working memory, among other skills, as a precursor and complement to working on math facts. The automaticity enabled by increasing these skills, combined with the enhancement of concept formation should yield dramatically better results than an exclusive focus on math fact drills

Some of the core deficits described earlier in this document as the underlying factors contributing to the various subtypes of math disability are easily identified as the skills

that can be enhanced via cognitive skills therapy, especially working memory and processing speed. These are skills that are expressly developed in BrainWare Safari, a computer program that develops 41 cognitive skills in a video-game format. In addition, BrainWare Safari develops visual-spatial skills that seem to exemplify the third subtype of math disability. Furthermore, especially given the co-morbidity of math and other disabilities, one can postulate that other cognitive skills also come into play but have not been specifically identified as part of the disability.

Empirical Support for the Impact of Cognitive Skill Development

In two studies, enhancement of cognitive skills has improved performance on a test of math fluency. Helms and Sawtelle⁵ found that students who completed 11 weeks of using BrainWare Safari increased their performance on the math fluency subtest of the Woodcock-Johnson® III battery by 1 year and 7 months, compared to a control group that experienced a decline in performance by 4 months over the same time period. In a second unpublished study⁶, nine ESL students improved their performance on the same test by 1 year and 9 months following use of the program for 11 weeks.

Research results demonstrating the impact of cognitive skill development on other formative and standardized assessments of math performance are not available but are under way.

Implications

Improvement of basic cognitive skills holds significant promise for improving math ability. The potential for this approach is particularly attractive given the failure of traditional “drill and kill” approaches and the lack of other consistently effective and scalable interventions. Moreover, the presence of developmental delays need not become a barrier to academic progress when a short-term (e.g., 11-week) intervention is available. By developing the underlying cognitive skills that allow the brain to process more information with more automaticity, initial results are demonstrating that mathematical capacity and understanding can be developed to a much higher level in a relatively short period of time.

Learning Enhancement Corporation
200 S. Wacker Drive, Suite 3100
Chicago, IL 60606
877-Brain-10 (877-272-4610)
www.LearningEnhancement.com
www.MyBrainWare.com

⁵ Helms, Don, and Sawtelle, Sara M. “A Study of the Effectiveness of Cognitive Skills Therapy Delivered in a Video-Game Format”. *Optom Vis Dev* 38.1(2007)19-26.

⁶ Unpublished results with group of 9 children from an Asian Community center in Naperville IL.